Soil Spatial Variability Effect on Soil Structure Interaction Studies: Enveloping Uncertainties in Structural Response

Nicholas Simos,[a] and Carl J. Costantino[a]

This paper addresses the effect that soil spatial variability may have on the seismic response of a structure. Using a parametric study, a probabilistic model enabling the enveloping of uncertainties associated with the soil-structure-interaction component of the seismic problem is formulated. The effects of most-likely sources of uncertainty, such as variability of “distinct” soil layer profile and variability of controlling soil properties, are addressed with a probabilistic profile in which randomization of key parameters that appear to have the most impact on the results of deterministic analyses is implemented. This is achieved through the use of stochastic finite elements along with the introduction of correlation functions. The primary goals are (a) the formulation of a mathematical process and modeling that connects uncertainty in the soil properties and profile with SSI, and (b) the enveloping of uncertainties in characterizing a site for which soil data are scarce, and (c) the utilization of real data to generate best estimates of statistical parameters used in the probabilistic description of soil systems. As a working example of assessing the effects of soil variability and uncertainty, an existing nuclear facility resting on not-so-well characterized soil is examined.

INTRODUCTION

This paper attempts to formulate an engineering approach that can deal with issues in soil-structure interaction of critical facilities, such as nuclear power plants, that come about from significant spatial fluctuations of key properties of the foundation soil including limitations in the available “in-situ” data.

It is well known that soil properties exhibit variations even within an otherwise “homogeneous” profiles or distinct subsurface layers (Fig. 1). Some of the variation can be

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[a] Brookhaven National Laboratory, Upton, NY 11973

traced to the depth location and to the influence of overburden. However, natural property variation combined with the limitations of characterizing the soil within optimal volumes (optimal volume would be considered such that the properties are fully characterized and essentially remain constant) will inevitably lead to situations where there exists a significant fluctuation of property values. The problem tends to become more acute when the subsurface is composed of “distinct” layers with even greater variability in the properties between them. All of the above are further complicated with the fact that either layer thickness or the bedrock depth vary spatially.

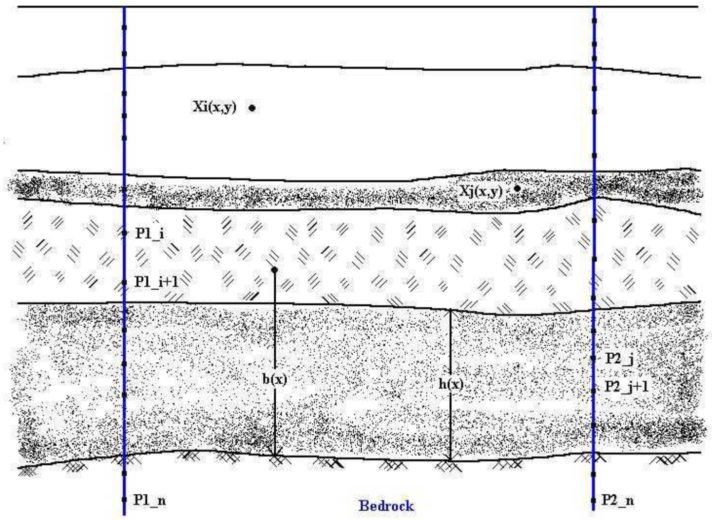

Figure 1. Typical layered profile with limited soil property

In typical deterministic analyses, average or equivalent properties for either the sub-layers or the whole soil deposit are being used. Layered deposits are studied considering layers of constant thickness throughout the domain of interest. The fundamental question, from the engineering point of view, is how do these real spatial fluctuations in both the soil properties and layering profile affect the response of a critical facility resting on the soil and how does one go about incorporating these uncertainties into its response. The two issues that need to be evaluated are (a) how does natural variability in site properties influence structural seismic response and (b) how should the input ground motion be characterized to properly evaluate this response. In this study the first issue is discussed.

A simple way used in addressing the question is the introduction of equivalent rather than average values of the soil parameters. A more “exotic” approach is the use of probabilistic/stochastic models that treat variability through complex mathematical relations.

While the latter may not seem very practical for wider implementation in real system studies, advances in finite element techniques and availability of computing capacity, have made it possible to be considered as an option in studying complex soil foundation formations. Both approaches, however, would require a minimum amount of “real” data from the site in order to build valid statistical models. In Baecher (1979) means of establishing “equivalent” parameters as well as how they may be linked to deterministic analyses are discussed. The importance of the character of the spatial variation rather than the “trend” of the mean value is examined. A more extensive discussion of probabilistic modeling of soil profiles is presented in Vanmarcke (1977). Specifically, correlation functions between soil property values at different locations and measures of fluctuation for incorporation into a randomized soil profile are generated. This study represents a serious attempt to link probabilistic models with finite element representations of the soil while focusing on soil settlement issues. While focusing on static loads of deterministic nature Yamazaki (1988) attempts, through the use of finite element formulation, to address structural response of a system with spatial property variability. Probabilistic models of soil profiles with emphasis on the variability of layer thickness were generated in Toro (1997). In Simos (1996) finite element techniques were used to address the response of a structure to probabilistic/stochastic loads rather than probabilistic description of the structure itself.

The variability in site properties was investigated to significant detail at the Savannah River Site, Toro (1997) where a large number of deep borings, cone penetrometers and suspension logger profiles were taken to evaluate shear wave velocity properties. Approximately 200 interpreted shear wave velocity profiles were developed from this data. The site is a deep soil site, extending to depths varying from 600' to over 1000' and is underlain by relatively hard rocks. Rock shear wave velocities typically were of the order of 8,000 fps to 10,000 fps. From this data, a probabilistic model was constructed to define best estimate values and variation of shear wave velocity profiles across the site as well as a variation in stratigraphy across the site. The model consists of three components, namely, (a) a model that describes the random stratigraphy at the site, (b) a model that describes a median site velocity profile and (c) a model that describes potential deviations in velocity of each layer from the median and its correlation with the velocity in the layer above. The model is typically used to generate artificial velocity profiles which are used to generate appropriate frequency dependent site amplification factors to define surface ground motion spectra.

PROBABILISTIC SOIL MODEL

The probabilistic model and its subsequent representation into the workings of the finite element procedure is based on the following approach. It is assumed that spatial variation in a property, such as shear wave velocity, exists within an identified soil layer. Interfaces between layers may be assumed as known from acoustic contrast (i.e. geophysical studies). A best estimate value of the property may exist from in-situ exploration at a number of locations within the layer but not enough for a complete description. A standard deviation may also be derived from these data. The influence of nonlinear behavior of soil and the effect of free-field response on these properties also needs to be incorporated into these evaluations.

The variable property (e.g. shear wave velocity) is assumed to be a 2-D homogeneous stochastic process and it is described by the mean and an added fluctuation random function $\varphi(x,y)$ with zero mean. In the finite element idealization of the site it is assumed that $\varphi(x,y)$ remains constant over its area/volume and it randomly varies between elements. Thus, with **N** finite elements over a region (layer) where variation is anticipated, a vector of size **N** is generated. The **N** random realizations of the property fluctuation are treated as normally distributed deviates with zero mean and variance equal to unity (times an expected range of fluctuation indicated by the standard deviation of the real data). The random (Gaussian) realizations are further assumed to exhibit correlation characteristics with the neighboring elements that typically weaken with distance. This is expected to be the case in real soils where fluctuations in soil properties are not expected to be dramatic between neighboring locations except in the case of a distinct layer interface. The scale of fluctuation measuring the distance within which there is strong correlation can be considered as one of the parameters of the probabilistic model and can only be based on actual soil data. The correlation between locations within the site is expressed in the form of a covariance matrix that links the random realizations of the property in question (i.e. shear velocity) between the two locations. The analytical model describing the probabilistic treatment of the soil that was implemented into the finite element analysis is outlined below.

Consider the fluctuating component $\varphi(x,y)$ of a soil property exhibiting spatial variability over a domain that is assumed to have zero mean

$$E[\phi(x)] = 0 \tag{1}$$

and auto-correlation function

$$R_{\varphi\varphi}(\xi) = E[\varphi(x)\varphi(x+\xi)] \tag{2}$$

where ***x*** is the position vector and ***ξ*** is the separation vector between two locations.

To introduce a correlation of the random property (i.e., shear velocity) between two locations in the deposit separated by a vector ***ξ*,** the auto-correlation function may be used. Eqns. 3a and 3b depict two suggested correlation forms

$$R_{\varphi\varphi}(\xi) = \exp\left[-\left(\frac{\xi}{\delta}\right)^2\right] \tag{3a}$$

$$R_{\varphi\varphi}(\xi) = \exp\left[-\frac{\xi}{\delta}\right]\left[1+\frac{\xi}{\delta}\right] \tag{3b}$$

where δ defines the distance of strong correlation of the soil property from point to point. Detailed studies of real soils are needed to establish a realistic basis for this parameter.

To formulate the correlation of values between locations $\boldsymbol{x_i}$ and $\boldsymbol{x_j}$ (centroid of two elements) in the probabilistic model, the covariance matrix is utilized. Specifically, the two locations, and for randomized field ***φ*** of size N (equal to the number of finite elements), have assumed a value of the property defined as $\boldsymbol{\varphi_i}$ and $\boldsymbol{\varphi_j}$. The covariance matrix of Eqn. 4 can be used to generate a random vector ***λ*** (also of size N) expressing the correlation that exists between the randomized property in two locations.

$$Cov[\varphi_i \varphi_j] = R_{\varphi\varphi}\{\xi_{ij}\} \tag{4}$$

$$\{\lambda\}^T = [L]\{\varphi\}^T \tag{5}$$

where ***L*** is the lower triangular matrix resulting from the Cholesky decomposition of the covariance matrix $\boldsymbol{C_{\varphi\varphi}}$ satisfying the following relationship,

$$LL^T = C_{\varphi\varphi} \tag{6}$$

Thus, starting from a vector consisting of N independent random realizations of the property fluctuation (as many as the finite elements in the zone of interest) and using a decomposition technique of the covariance matrix, a vector that contains both the randomness and the correlation with the other elements is formed. This vector of values is added to the expected value of the property for the zone leading to a distribution of values that are correlated. Using a Monte Carlo approach, ***M*** random vectors ***φ*** are generated leading (through the covariance matrix and its decomposition) to ***M*** profiles expressed in the form of

λ vectors. The computational cost of generating random correlated fields is minimized due to the fact the decomposition of the covariance matrix only takes place once. With M realizations of the property exhibiting spatial variation, M solutions of the seismic problem are derived through the Monte Carlo approach. Therefore, statistics on the response of interest, based on the M distinct solutions of the problem, can be made and the effect of the spatial variability can be addressed.

The formulation described has been implemented into the finite element code POROSLAM (Simos, et al., 1996). Also, a technique that incorporates actual values of the property, that may exist at a few locations and are not enough to make a statistical sample describing the whole site) has been implemented.

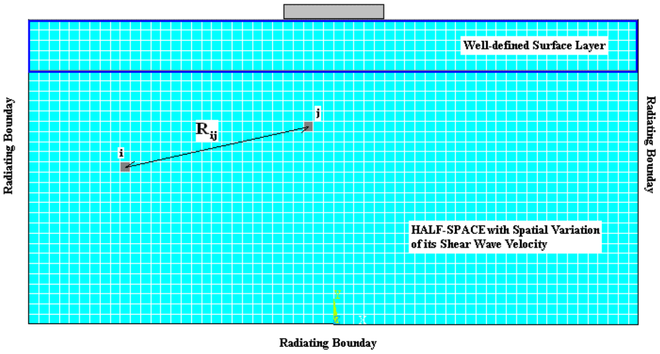

Figure 2. Idealized surface layer overlaying a half space exhibiting spatial variability in the shear wave velocity

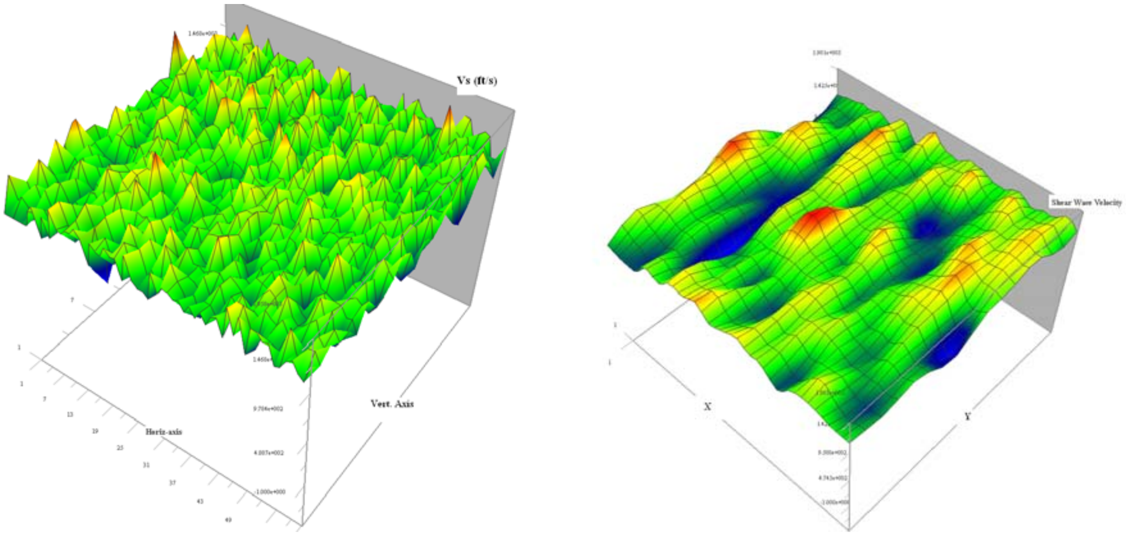

Figure 3. (a) Randomized shear wave velocity in half space domain of Figure 2 without special correlation and (b) resulting shear wave spatial distribution after cross-correlation procedures have been introduced in the randomized field shown in (a).

The probabilistic formulation has been used to evaluate soil compliances of an idealized site in which a well-defined surface layer is overlaying a half space with spatial variability of the shear wave velocity. Figure 2 depicts the finite element model utilized. Figures 3a shows the spatial distribution of the shear wave velocity generated as a random process without any correlation. Figure 3b is the resulting distribution of shear wave velocity after Equation 5 has been applied. Figures 4a and 4b depict the vertical and rotational compliances using the expected (or mean) shear wave velocity in the half space as well as the two extremes of the assumed distribution (± 3σ).

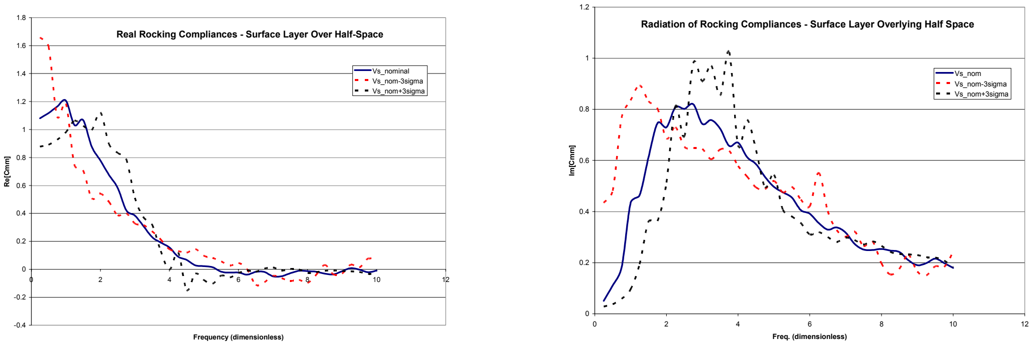

Figure 4. (a) Rocking compliances (real) of the idealized model of Figure 2 computed for the expected shear wave value in a homogeneous half-space and the two extremes in the distribution of the assumed fluctuation, and (b) rocking compliances (imaginary) for the same case as in (a).

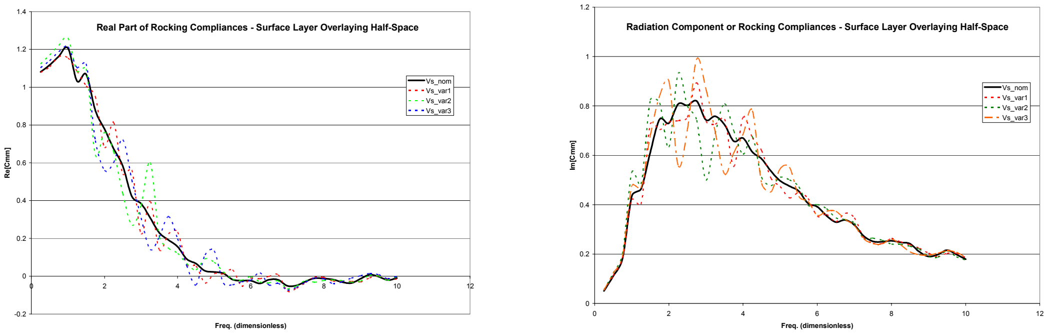

Figure 5. (a) Rocking compliances (real) of the idealized model of Figure 2 computed for the expected shear wave value in a homogeneous half-space and several idealizations of the random distribution describing shear wave fluctuation, and (b) rocking compliances (imaginary) for the same case as in (a).

Figures 5a and 5a depict the same compliances generated by considering various random realizations of the shear wave velocity in the half space. It is apparent that the probabilistic treatment leads to a solution in which the mean of the response generated with the Monte Carlo approach approaches the response resulting from using the expected value of the property over the region considered to exhibit spatial variability. On the other hand, the use of the extreme values of the distribution as means of generating an envelope of the response leads to significantly different results, as compared to the expected value, over certain range of frequency of the dynamic analysis.

ARMENIAN NPP SITE – A WORKING EXAMPLE

The possibility that the local site conditions play a more significant role than had been assumed thus far remains as one of the open issues in the NPP seismic re-evaluation effort especially as it relates to soil-structure interaction.

Initial geo-technical studies, conducted prior to the plant construction, revealed the presence of soil layers near the surface with uncharacteristically low shear wave velocities. Velocities measured in the soils of these layers, sandwiched between basalts, were estimated to be as low as 300m/s. Figure 6 depicts the soil profile near the surface. Limited number of borehole data obtained prior to plant's construction (augmented by additional data during re-evaluation) provided a basic profile of the subsurface and average soil property values. Recent geophysical studies, however, contradicted the original assessment of uncharacteristically low shear wave velocities in the layers near the surface leading to further uncertainty in the available data. As a result, several seismic-related questions continue to require answers especially those connected to SSI and subsequently to the generation of floor response spectra. In an effort to make the best "guesstimate" site response analyses have been performed using the SHAKE code and addressed the motion convolution issues.

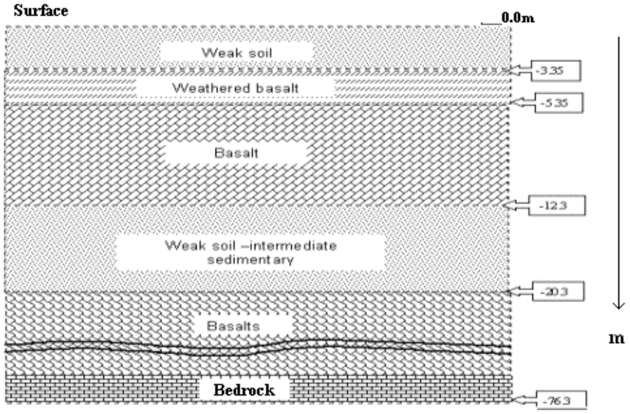

Figure 6. Layered soil profile under the Armenian Nuclear Power Plant (ANPP)

To establish an envelope that would enclose these two extremes and provide a basis for variability in the response of the reactor facility, two profiles have been recommended for use in the seismic motion convolution, SSI and structural response spectra. These profiles are depicted on Figure 7.

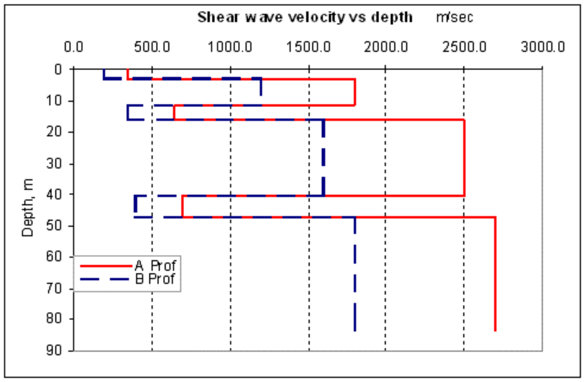

Figure 7. Adopted soil profiles for the ANPP site for deterministic SSI studies

Studies based on the SASSI code for 3-D description of the overall system (subsurface and reactor building) and POROSLAM for a detailed 2-D representation of the subsurface (foundation and subsurface interaction) were performed and assessed the variability in the response (either in terms of foundation impedances or structural seismic spectra). Results depicted in Figures 8 and 10 indicate that there is a significant effect, attributed to the choice of soil profile. Figure 10a and 10b depict the variation in response (foundation impedances) that may result from using an *equivalent* profile for the entire soil deposit. While for small values of frequency the *equivalent* soil approach appears to be acceptable, for higher values of frequency the solution deviates significantly.

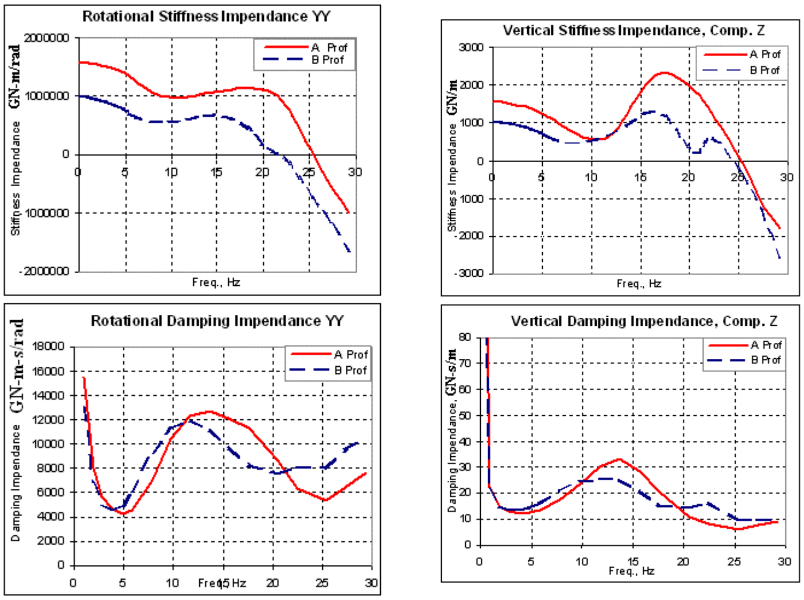

Figure 8. (a) Rotational ANPP impedances (real and imaginary) generated with a 3-D SASSI model and (b) vertical ANPP impedances

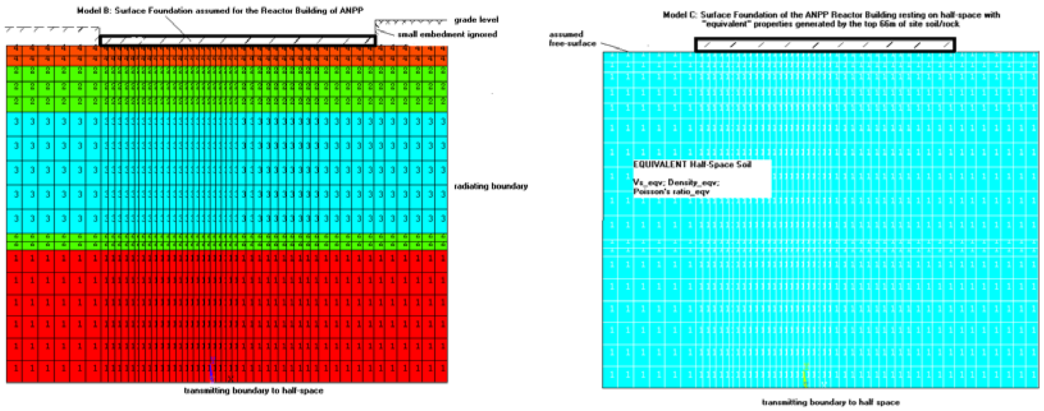

Figure 9. (a) Idealized model of the ANPP soil profile in 2-D SSI analyses using the POROSLAM code (Simos, *et al*) and (b) idealization of the ANPP site based on an *equivalent* half space.

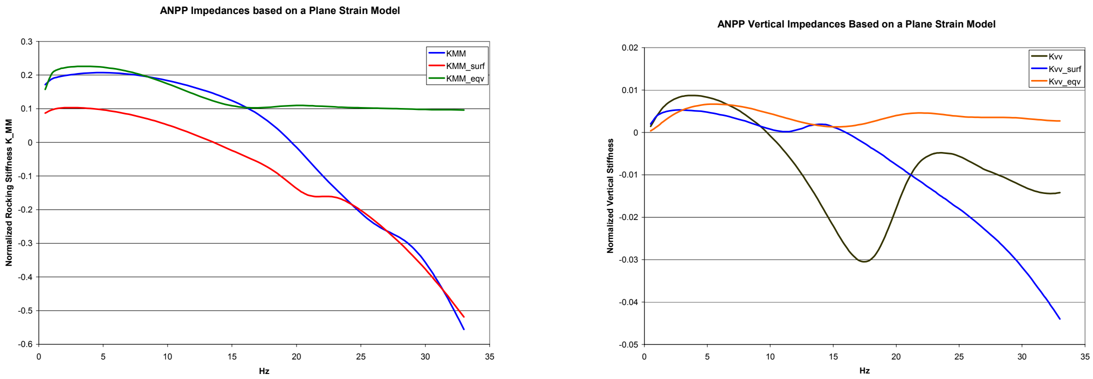

Figure 10. (a) Rocking stiffness resulting from the idealized models of Figure 9 and (b) vertical stiffness. Also shown is the stiffness resulting from a model of the ANPP site that closely describes the particular foundation design of the nuclear facility (shown as K_{MM} and K_{VV}).

SUMMARY

Based on the preliminary assessment of the seismic response of a real nuclear structure resting on soil with incomplete information on the key properties, and the difficulties encountered in establishing a representative profile for the analysis, a probabilistic approach is being introduced to enable the enveloping of the uncertainties in the overall response. It is further apparent from the studies that "equivalent" representation of the deposit does not lead to an acceptable solution throughout the frequency range of interest.

By utilizing principles of probabilistic description of soil that have been tested in previous studies as well as finite element procedures a probabilistic model based on Monte Carlo has been generated. The probabilistic model enables for properties in several layers of the soil deposit to be treated as random with various degrees of correlation between locations within the layer. The mechanics of the process have been implemented into a finite element code in order to assess how the variability in a property, such as shear wave velocity, can affect the structural response. The enveloping process of the uncertainty is generated by the response statistics.

To gain a better understanding of how (a) "homogeneous" soil properties within a deposit vary and (b) how one can arrive at better estimates of statistical parameters associated with a probabilistic model for a certain type of soil, sets of actual data are being examined.

As pointed out, an important component of the overall problem is the characterization of the ground motion and the way it may influence the response in conjunction with the soil variability needs to be examined.

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